AI -1544

M. A./M. Sc. (Previous)
Term End Examination, 2020-21

MATHEMATICS

Paper : Fifth

(Advanced Discrete Mathematics)

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 36

*Note : Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.*

1. (a) Define and explain each of the following :
   (i) Conditional and Biconditional statements
   (ii) Converse, inverse and contrapositive of $p \to q$.

   (iii) Equivalent statement.
   (iv) De Morgan's laws.

(b) Prove that :

$$\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q))$$

without constructing truth table.

2. (a) Define arguments. Modus ponens and law of syllogism.
   Prove that the following arguments is valid :

$$p$$
$$p \to q$$
$$q$$

(b) (i) Simplify the following :

(I) $(P \wedge Q) \wedge \sim P$

(II) $\sim(\sim P \wedge Q) \wedge (\sim P \vee Q) \wedge (P \vee Q)$

(ii) Using $\wedge$ and $\sim$ for A and N respectively rewrite the following statements :

(I) A N A P A q p A A N q r p

(II) A A p N r A q N p

3. (a) Define idempotent element of a semi group. Prove that every finite semigroup has an idempotent element.

   (b) Define Homomorphism of Monoids.
   Let (S, *) and (T, o) be semigroups. If $f : S \to T$ is a semigroup homorphism, then semi group (T, o) is isomorphic to some quotient semigroup of (S, *)

4. (a) Establish the equivalence of the two definitions of a lattice.

   (b) Explain Bounded lattices. Prove that every finite lattice is a bounded lattice.

5. (a) In any boolean Algebra, show that :

   (i) $(a+b)(b+c)(c+a) = ab + bc + ca$

   (ii) $(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

   (b) (i) Express the following Boolean functions

   $f(x, y, z) = (x+y)(x+y')(x'+z)$

   in disjunctive normal form in three variables.

   (ii) In a Boolean Algebra, show that :

   $f(x, y) = xf(1, y) + x'f(0, y)$

6. (a) Use the Karnaugh map respresentation to find a minimal form of each of the following functions :

   (i) $f(x, y) = x'y + xy$

   (ii) $f(x, y, z) = xyz + xy'z + x'yz + x'y'z$

   (b) (i) Draw the logic circuit for the following expression :

   $x \cdot y + zy'$

   (ii) Draw the logic circuit with inputs a, b, c and output f where

   $f = ab'c + abc' + ab'c'$

7. (a) Explain phrase structure grammer. Find the language $L(G)$ over $A = \{a, b, c\}$ genereated by the grammer G with production

   $S \to aSb$, $aS \to Aa$, $Aab \to C$

   (b) What is polish notation. Explain conversion of infix experience to polish notation.

8. (a) Design a finite state machine M which can add two binary numbers.

(b) Define equivalent machine. Construct the state diagram for the finite state machine with the state table as given below :

State	f input		g output	
	0	1	0	1
S_0	S_1	S_0	1	0
S_1	S_2	S_1	0	1
S_2	S_3	S_1	1	1
S_3	S_2	S_1	0	0

9. (a) Define the following :

(i) Adjacent vertex

(ii) Degree vertex

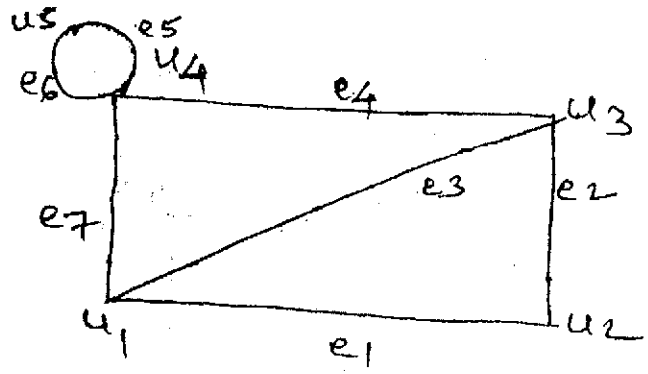
(iii) Isolated vertex

(iv) Pendent vertex

(v) Regular graphs

(b) Define In-degree and out-degree, Adjacency matrix, Incidence matrix. Write the incidence

matrix of the following graph :



10. (a) Define spanning tree with example. Define Branch and chord of a spanning tree. Prove that every connected graph has atleast one spanning tree.

(b) Define cut set with example. Write properties of cut sets. If every region of a simple planer graph with n vertices and e edges is bounded by k edges, show that :

$$e = \frac{k(n-2)}{(k-2)}$$